Chapter 3

Cash Flow Engineering and Forward Contracts

1. Introduction

All financial instruments can be visualized as bundles of cash flows. They are designed so that market participants can trade cash flows that have different *characteristics* and different *risks*. This chapter uses *forwards* and *futures* to discuss how cash flows can be replicated and then repackaged to create synthetic instruments.

It is easiest to determine replication strategies for linear instruments. We show that this can be further developed into an analytical methodology to create synthetic equivalents of complicated instruments as well. This analytical method will be summarized by a (contractual) *equation*. After plugging in the right instruments, the equation will yield the synthetic for the cash flow of interest. Throughout this chapter, we assume that there is *no default risk* and we discuss only *static replication methods*. Positions are taken and kept unchanged until expiration, and require no rebalancing. Dynamic replication methods will be discussed in Chapter 7. Omission of default risk is a major simplification and will be maintained until Chapter 5.

2. What Is a Synthetic?

The notion of a synthetic instrument, or replicating portfolio, is central to financial engineering. We would like to understand how to price and hedge an instrument, and learn the risks associated with it. To do this we consider the cash flows generated by an instrument during the lifetime of its contract. Then, using other *simpler*, *liquid* instruments, we form a portfolio that replicates these cash flows exactly. This is called a *replicating portfolio* and will be a synthetic of the original instrument. The constituents of the replicating portfolio will be easier to price, understand, and analyze than the original instrument.

In this chapter, we start with synthetics that can be discussed using *forwards* and *futures* and *money market products*. At the end we obtain a *contractual equation* that can be algebraically manipulated to obtain solutions to practical financial engineering problems.

2.1. Cash Flows

We begin our discussion by defining a simple tool that plays an important role in the first part of this book. This tool is the graphical representation of a cash flow.

By a cash flow, we mean a payment or receipt of cash at a specific *time*, in a specific *currency*, with a certain *credit risk*. For example, consider the default-free cash flows in Figure 3-1. Such figures are used repeatedly in later chapters, so we will discuss them in detail.

EXAMPLE:

In Figure 3-1a we show the cash flows generated by a default-free loan. Multiplying these cash flows by -1 converts them to cash flows of a deposit, or depo. In the figure, the horizontal axis represents time. There are two time periods of interest denoted by symbols t_0 and t_1. The t_0 represents the time of a USD100 cash inflow. It is shown as a rectangle above *the line. At time t_1, there is a cash out*flow, since the rectangle is placed below *the line and thus indicates a debit. Also note that the two cash flows have different sizes.*

We can interpret Figure 3-1a as cash flows that result when a market participant borrows USD100 at time t_0 and then pays this amount back with interest as USD105, where the interest rate applicable to period $[t_0, t_1]$ is 5% and where $t_1 - t_0 = 1$ year.

Every financial transaction has at least two counterparties. It is important to realize that the top portion of Figure 3-1a shows the cash flows from the borrower's point of view. Thus, if we look at the same instrument from the lender's point of view, we will see an inverted image of these cash flows. The lender lends USD100 at time t_0 and then receives the principal and interest at time t_1. The bid-ask spread suggests that the interest is the asking rate.

Finally, note that the cash flows shown in Figure 3-1a do not admit any uncertainty, since, both at time t_0 and time-t_1 cash flows are represented by a single rectangle with
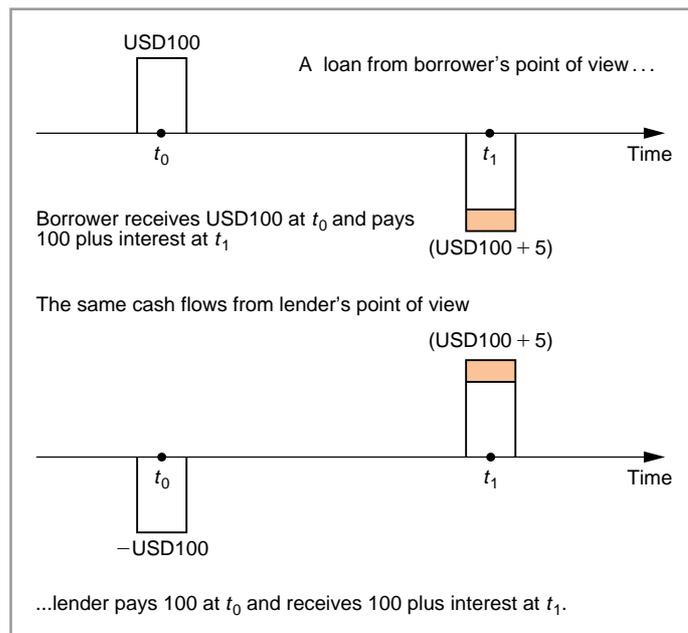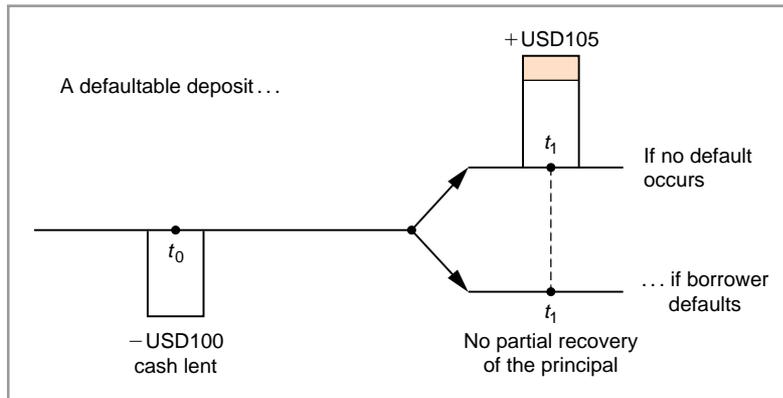

FIGURE 3-1a

FIGURE 3-1b

known value. If there were uncertainty about either one, we would need to take this into account in the graph by considering different states of the world. For example, if there was a default *possibility on the loan repayment, then the cash flows would be represented as in Figure 3-1b. If the borrower defaulted, there would be no payment at all. At time* t_1*, there are* two *possibilities. The lender either receives USD105 or receives nothing.*

Cash flows have special characteristics that can be viewed as *attributes*. At all points in time, there are market participants and businesses with different needs in terms of these attributes. They will exchange cash flows in order to reach desired objectives. This is done by trading financial contracts associated with different cash flow attributes. We now list the major types of cash flows with well-known attributes.

2.1.1.   Cash Flows in Different Currencies

The first set of instruments devised in the markets trade cash flows that are identical in every respect except for the *currency* they are expressed in.

In Figure 3-2, a decision maker pays USD100 at time t_0 and receives $100e_{t_0}$ units of Euro at the *same* time. This a *spot FX deal*, since the transaction takes place at time t_0. The e_{t_0} is the *spot* exchange rate. It is the number of Euros paid for one USD.

2.1.2.   Cash Flows with Different Market Risks

If cash flows with different market risk characteristics are exchanged, we obtain more complicated instruments than a spot FX transaction or deposit. Figure 3-3 shows an exchange of
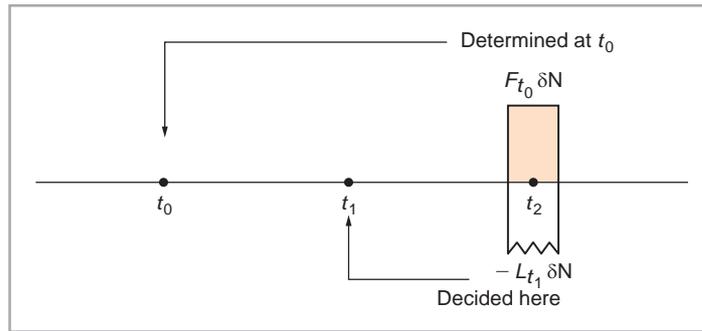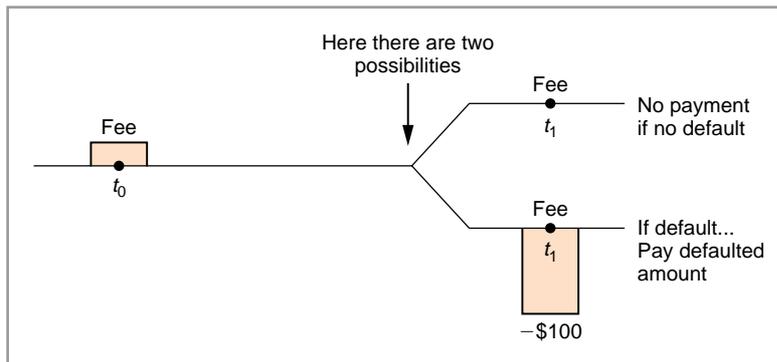

FIGURE 3-2

FIGURE 3-3

FIGURE 3-4

cash flows that depend on different market risks. The market practitioner makes a payment proportional to L_{t_1} percent of a *notional amount* N against a receipt of F_{t_0} percent of the same N. Here L_{t_1} is an unknown, *floating* Libor rate at time t_0 that will be learned at time t_1. The F_{t_0}, on the other hand, is set at time t_0 and is a *forward* interest rate. The cash flows are exchanged at time t_2 and involve two different types of risk. Instruments that are used to exchange such risks are often referred to as *swaps*. They exchange a *floating* risk against a *fixed* risk. Swaps are not limited to interest rates. For example, a market participant may be willing to pay a floating (i.e., to be determined) oil price and receive a fixed oil price. One can design such *swaps* for all types of commodities.

2.1.3.   Cash Flows with Different Credit Risks

The probability of default is different for each borrower. Exchanging cash flows with different credit risk characteristics leads to *credit instruments.*

In Figure 3-4, a counterparty makes a payment that is contingent on the *default* of a decision maker against the *guaranteed* receipt of a fee. Market participants may buy and sell such cash flows with different credit risk characteristics and thereby adjust their credit exposure. For example, AA-rated cash flows can be traded against BBB-rated cash flows.

2.1.4.   Cash Flows with Different Volatilities

There are instruments that exchange cash flows with different volatility characteristics. Figure 3-5 shows the case of exchanging a fixed volatility at time t_2 against a realized (floating) volatility observed during the period, $[t_1, t_2]$. Such instruments are called *volatility* or *Vol*-swaps.
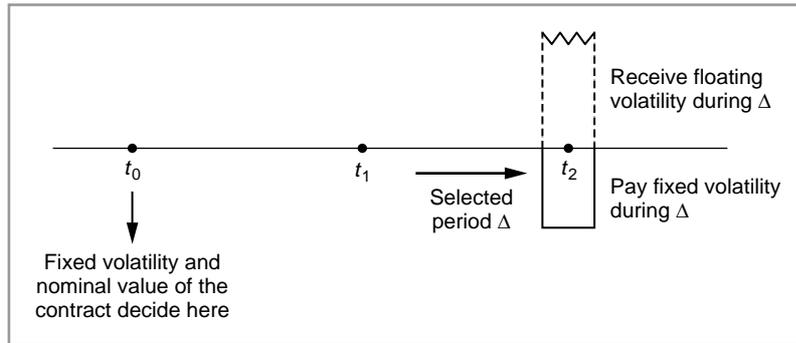
FIGURE 3-5

3.   Forward Contracts

Forwards, *futures contracts*, and the underlying interbank *money markets* involve some of the simplest cash flow exchanges. They are ideal for creating synthetic instruments for many reasons. Forwards and futures are, in general, linear permitting static replication. They are often very liquid and, in case of currency forwards, have homogenous underlying. Many technical complications are automatically eliminated by the homogeneity of a currency. Forwards and futures on interest rates present more difficulties, but a discussion of these will be postponed until the next chapter.

A forward or a futures contract can fix the future selling or buying price of an underlying item. This can be useful for hedging, arbitraging, and pricing purposes. They are essential in creating synthetics. Consider the following interpretation.

Instruments are denominated in different currencies. A market practitioner who needs to perform a required transaction in U.S. dollars normally uses instruments denoted in U.S. dollars. In the case of the dollar this works out fine since there exists a broad range of liquid markets. Market professionals can offer all types of services to their customers using these. On the other hand, there is a relatively small number of, say, *liquid* Swiss Franc (CHF) denoted instruments. Would the Swiss market professionals be deprived of providing the same services to their clients? It turns out that liquid Foreign Exchange (FX) forward contracts in USD/CHF can, in principle, make USD-denominated instruments available to CHF-based clients as well.

Instead of performing an operation in CHF, one can first buy and sell USD at t_0, and then use a USD-denominated instrument to perform any required operation. Liquid FX-Forwards permit *future* USD cash flows to be reconverted into CHF as of time t_0. Thus, entry into and exit from a different currency is fixed at the initiation of a contract. As long as liquid forward contracts exist, market professionals can use USD-denominated instruments in order to perform operations in any other currency without taking FX risk.

As an illustration, we provide the following example where a synthetic zero coupon bond is created using FX-forwards and the bond markets of another country.

EXAMPLE:

Suppose we want to buy, at time t_0, a USD-denominated default-free discount bond, with maturity at t_1 and current price $B(t_0, t_1)$. We can do this synthetically using bonds denominated in any other currency, as long as FX-forwards exist and the relevant credit risks are the same.

First, we buy an appropriate number of, say, Euro-denominated bonds with the same maturity, default risk, and the price $B(t_0, t_1)^E$. This requires buying Euros against dollars in the spot market at an exchange rate e_{t_0}. Then, using a forward contract on

Euro, we sell forward the Euros that will be received on December 31, 2005, when the bond matures. The forward exchange rate is F_{t_0}.

The final outcome is that we pay USD now and receive a known amount of USD at maturity. This should generate the same cash flows as a USD-denominated bond under no-arbitrage conditions. This operation is shown in Figure 3-6.

In principle, such steps can be duplicated for any (linear) underlying asset, and the ability to execute forward purchases or sales plays a crucial role here. Before we discuss such operations further, we provide a formal definition of forward contracts.

A *forward* is a contract written at time t_0, with a commitment to accept delivery of (deliver) N units of the underlying asset at a future date t_1, $t_0 < t_1$, at the *forward price F_{t_0}*. At time t_0, nothing changes hands; all exchanges will take place at time t_1. The current price of the underlying asset S_{t_0} is called the spot price and is not written anywhere in the contract, instead, F_{t_0} is used during the settlement. Note that F_{t_0} has a t_0 subscript and is fixed at time t_0. An example of such a contract is shown in Figure 3-6.

Forward contracts are written between two parties, depending on the needs of the client. They are *flexible* instruments. The size of contract N, the expiration date t_1, and other conditions written in the contract can be adjusted in ways the two parties agree on.

If the same forward purchase or sale is made through a *homogenized contract*, in which the size, expiration date, and other contract specifications are preset, if the trading is done in a
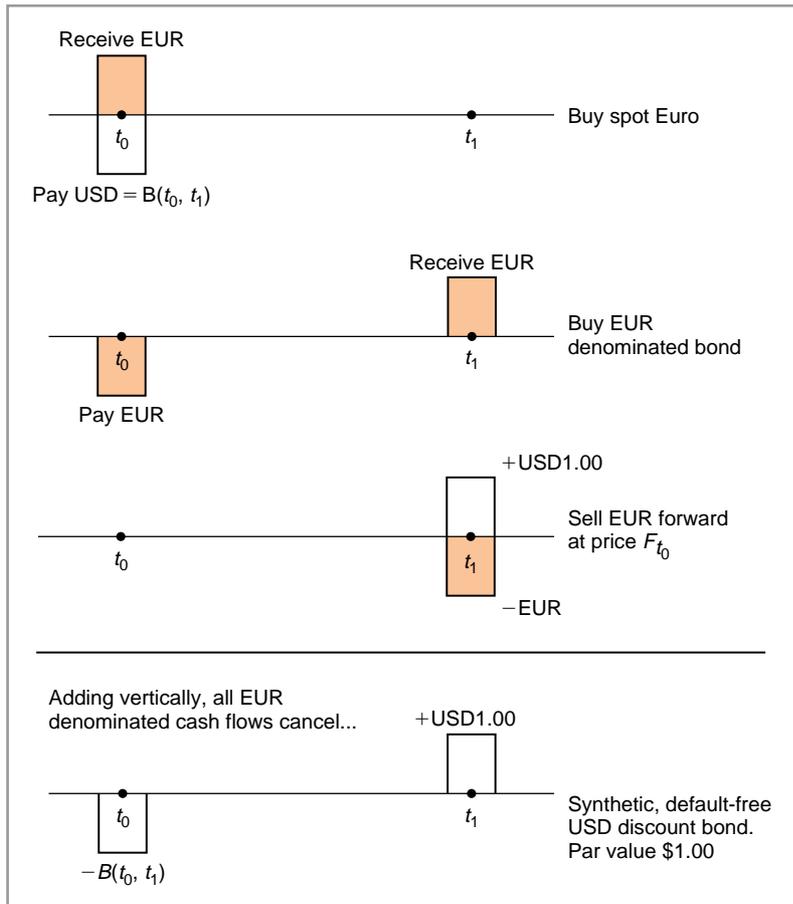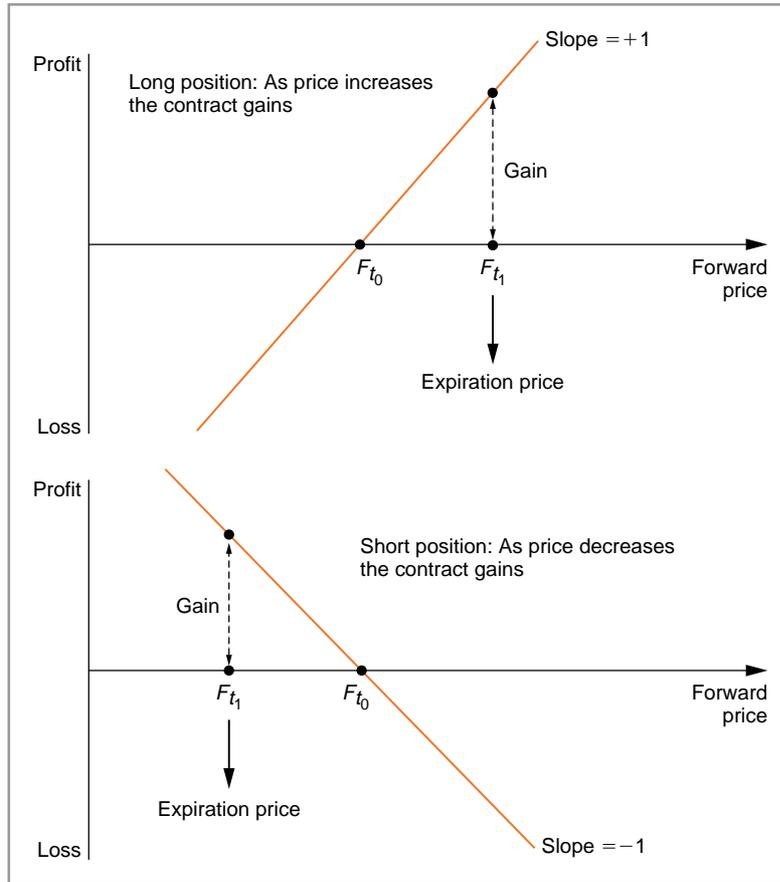

FIGURE 3-6

formal exchange, if the counterparty risk is transferred to a clearinghouse, and if there is formal *mark-to-market*, then the instrument is called *futures*.

Positions on forward contracts are either *long* or *short*. As discussed in Chapter 2, a *long position* is a commitment to accept delivery of the contracted amount at a future date, t_1, at price F_{t_0}. This is displayed in Figure 3-7. Here F_{t_0} is the contracted forward price. As time passes, the corresponding price on newly written contracts will change and at expiration the forward price becomes F_{t_1}. The difference, $F_{t_1} - F_{t_0}$, is the profit or loss for the position taker. Note two points. Because the forward contract does not require any cash payment at initiation, the time-t_0 value is *on* the x-axis. This implies that, at initiation, the market value of the contract is zero. Second, at time t_1 the spot price S_{t_1} and the forward price F_{t_1} will be the same (or very close).

A *short position* is a commitment to deliver the contracted amount at a future date, t_1, at the agreed price F_{t_0}. The short forward position is displayed in Figure 3-7. The difference $F_{t_0} - F_{t_1}$ is the profit or loss for the party with the short position.

EXAMPLES:

Elementary forwards and futures contracts exist on a broad array of underlyings. Some of the best known are the following:

1. *Forwards on* currencies. *These are called FX-forwards and consist of buying (selling) one currency against another at a future date t_1.*

2. *Futures on* loans and deposits. *Here, a currency is exchanged against itself, but at a later date. We call these* forward loans *or* deposits. *Another term for these is* forward-forwards. *Futures provide a more convenient way to trade interest rate commitments; hence, forward loans are not liquid. Futures on forward loans are among the most liquid.*

3. *Futures on commodities, e.g., be oil, corn, pork bellies, and gold. There is even a thriving market in futures trading on weather conditions.*

4. *Futures and forwards on individual stocks and* stock indices. *Given that one cannot settle a futures contract on an index by delivering the whole basket of stocks, these types of contracts are* cash settled. *The losers compensate the gainers in cash, instead of exchanging the underlying products.*

5. *Futures contracts on* swaps. *These are relatively recent and they consist of future swap rate commitments. They are also settled in cash. Compared to futures trading, the OTC forward market is much more dominant here.*

6. *Futures contracts on volatility indices.*

We begin with the engineering of one of the simplest and most liquid contracts; namely the currency *forwards*. The engineering and uses of forward *interest rate products* are addressed in the next chapter.

4.   Currency Forwards

Currency forwards are very liquid instruments. Although they are elementary, they are used in a broad spectrum of financial engineering problems.

Consider the EUR/USD exchange rate.[1] The cash flows implied by a forward purchase of 100 U.S. dollars against Euros are represented in Figure 3-8a. At time t_0, a contract is written for the forward purchase (sale) of 100 U.S. dollars against $100/F_{t_0}$ Euros. The settlement—that is to say, the actual exchange of currencies—will take place at time t_1. The forward exchange rate is F_{t_0}. At time t_0, nothing changes hands.

Obviously, the forward exchange rate F_{t_0} should be chosen at t_0 so that the two parties are satisfied with the future settlement, and thus do not ask for any immediate compensating payment. This means that the time-t_0 value of a forward contract *concluded* at time t_0 is *zero*. It may, however, become positive or negative as time passes and markets move.

In this section, we discuss the structure of this instrument. How do we create a synthetic for an instrument such as this one? How do we decompose a forward contract? Once this is understood, we consider applications of our methodology to hedging, pricing, and risk management.

A general method of engineering a (currency) forward—or, for that matter, any linear instrument—is as follows:

1. Begin with the cash flow diagram in Figure 3-8a.
2. Detach and carry the (two) rectangles representing the cash flows into Figures 3-8b and 3-8c.
3. Then, add *and* subtract new cash flows at carefully chosen dates so as to *convert* the detached cash flows into meaningful financial contracts that players will be willing to buy and sell.
4. As you do this, make sure that when the diagrams are added *vertically*, the newly added cash flows cancel out and the original cash flows are *recovered*.

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[1] Written as EUR/USD in this quote, the *base currency* is the Euro.

(a) A Forward contract

Receive USD100

t_0

t_1

Pay $(100/F_{t_0})$EUR

This can be decomposed into two cash flows . . .

(b)

t_0

t_1

Pay $(100/F_{t_0})$EUR
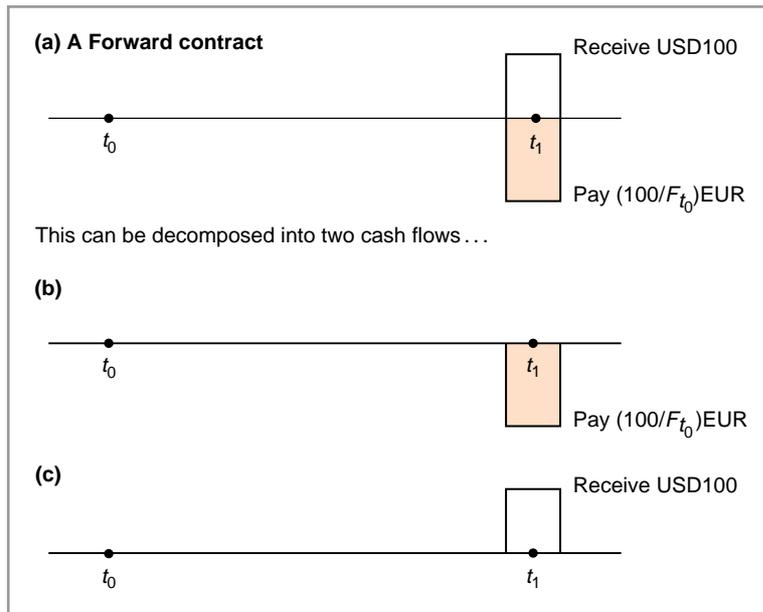
(c)

Receive USD100

t_0

t_1

FIGURE 3-8abc

This procedure will become clearer as it is applied to progressively more complicated instruments. Now we consider the details.

4.1. Engineering the Currency Forward

We apply this methodology to engineering a currency forward. Our objective is to obtain a contractual equation at the end and, in this way, express the original contract as a *sum* of two or more elementary contracts. The steps are discussed in detail.

Begin with cash flows in Figure 3-8a. If we detach the two cash flows, we get Figures 3-8b and 3-8c. At this point, nobody would like to buy cash flows in Figure 3-8b, whereas nobody would sell the cash flows in Figure 3-8c. Indeed, why pay something without receiving anything in return? So at this point, Figures 3-8b and 3-8c cannot represent tradeable financial instruments.

However, we can *convert* them into tradeable contracts by inserting new cash flows, as in step 3 of the methodology. In Figure 3-8b, we *add* a corresponding cash *in*flow. In Figure 3-8c we add a cash *out*flow. By adjusting the size and the timing of these new cash flows, we can turn the transactions in Figures 3-8b and 3-8c into meaningful financial contracts.

We keep this as simple as possible. For Figure 3-8b, add a positive cash flow, preferably at time t_0.[2] This is shown in Figure 3-8d. Note that we denote the size of the newly added cash flow by $C_{t_0}^{\mathrm{EUR}}$.

In Figure 3-8c, add a negative cash flow at time t_0, to obtain Figure 3-8e. Let this cash flow be denoted by $C_{t_0}^{\mathrm{USD}}$. The size of $C_{t_0}^{\mathrm{USD}}$ is not known at this point, except that it has to be in USD.

The *vertical addition* of Figures 3-8d and 3-8e should replicate what we started with in Figure 3-8a. At this point, this *will* not be the case, since $C_{t_0}^{\mathrm{USD}}$ and $C_{t_0}^{\mathrm{EUR}}$ do not cancel out at time t_0 as they are denominated in different currencies. But, there is an easy solution to this. The "extra" time t_0 cash flows can be eliminated by considering a third component for the synthetic.

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[2] We *could* add it at another time, but it would yield a more complicated synthetic. The resulting synthetic will be less liquid and, in general, more expensive.

Consider Figure 3-8f where one exchanges $C_{t_0}^{\text{USD}}$ against $C_{t_0}^{\text{EUR}}$ at time t_0. After the addition of this component, a vertical sum of the cash flows in Figures 3-8d, 3-8e, *and* 3-8f gives a cash flow pattern identical to the ones in Figure 3-8a. If the credit risks are the same, we have succeeded in replicating the forward contract with a synthetic.

4.2. *Which Synthetic?*

Yet, it is still not clear what the synthetic in Figures 3-8d, 3-8e, and 3-8f consists of. True, by adding the cash flows in these figures we recover the original instrument in Figure 3-8a, but what kind of contracts do these figures represent? The answer depends on how the synthetic instruments shown in Figures 3-8d, 3-8e, and 3-8f are interpreted.

This can be done in many different ways. We consider two major cases. The first is a deposit-loan interpretation. The second involves Treasury bills.

4.2.1. A Money Market Synthetic

The first synthetic is obtained using money market instruments. To do this we need a brief review of money market instruments. The following lists some important money market instruments, along with the corresponding quote, registration, settlement, and other conventions that will have cash flow patterns similar to Figures 3-8d and 3-8e. The list is not comprehensive.
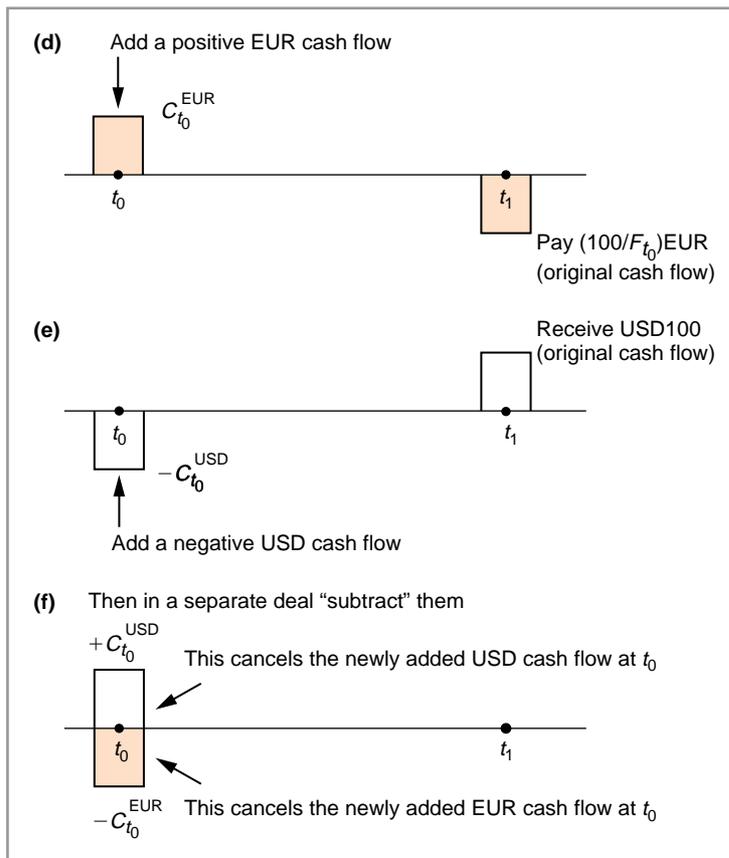

FIGURE 3-8def

EXAMPLE:

Deposits/loans. *These mature in less than 1 year. They are denominated in domestic and Eurocurrency units. Settlement is on the* same day *for domestic deposits and in 2 business days for Eurocurrency deposits. There is no registration process involved and they are not negotiable.*

Certificates of deposit (CD). *Generally these mature in up to 1 year. They pay a coupon and are sometimes sold in discount form. They are quoted on a yield basis, and exist both in domestic and Eurocurrency forms. Settlement is on the same day for domestic deposits and in 2 working days for Eurocurrency deposits. They are usually bearer securities and are negotiable.*

Treasury bills. *These are issued at 13-, 26-, and 52-week maturities. In France, they can also mature in 4 to 7 weeks; in the UK, also in 13 weeks. They are sold on a discount basis (U.S., UK). In other countries, they are quoted on a yield basis. Issued in domestic currency, they are bearer securities and are negotiable.*

Commercial paper (CP). *Their maturities are 1 to 270 days. They are very short-term securities, issued on a discount basis. The settlement is on the same day, they are bearer securities, and are negotiable.*

Euro-CP. *The maturities range from 2 to 365 days, but most have 30- or 180-day maturities. Issued on a discount basis, they are quoted on a yield basis. They can be issued in any Eurocurrency, but in general they are in Eurodollars. Settlement is in 2 business days, and they are negotiable.*

How can we use these money market instruments to interpret the synthetic for the FX-forward shown in Figure 3-8?

One money market interpretation is as follows. The cash flow in Figure 3-8e involves making a payment of $C_{t_0}^{\mathrm{USD}}$ at time t_0, to receive USD100 at a later date, t_1. Clearly, an interbank *deposit* will generate exactly this cash flow pattern. Then, the $C_{t_0}^{\mathrm{USD}}$ will be the present value of USD100, where the discount factor can be obtained through the relevant Eurodeposit rate.

$$C_{t_0}^{\mathrm{USD}} = \frac{100}{1 + L_{t_0}^{\mathrm{USD}}(\frac{t_1 - t_0}{360})} \tag{1}$$

Note that we are using an $ACT/360$-day basis for the deposit rate $L_{t_0}^{\mathrm{USD}}$, since the cash flow is in Eurodollars. Also, we are using money market conventions for the interest rate.[3] Given the observed value of $L_{t_0}^{\mathrm{USD}}$, we can numerically determine the $C_{t_0}^{\mathrm{USD}}$ by using this equation.

How about the cash flows in Figure 3-8d? This can be interpreted as a *loan* obtained in interbank markets. One receives $C_{t_0}^{\mathrm{EUR}}$ at time t_0, and makes a Euro-denominated payment of $100/F_{t_0}$ at the later date t_1. The value of this cash flow will be given by

$$C_{t_0}^{\mathrm{EUR}} = \frac{100/F_{t_0}}{1 + L_{t_0}^{\mathrm{EUR}}(\frac{t_1 - t_0}{360})} \tag{2}$$

where the $L_{t_0}^{\mathrm{EUR}}$ is the relevant interest rate in euros.

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[3] We remind the reader that if this was a domestic or eurosterling deposit, for example, the day basis would be 365. This is another warning that in financial engineering, conventions matter.

Finally, we need to interpret the last diagram in 3-8f. These cash flows represent an exchange of $C_{t_0}^{\text{USD}}$ against $C_{t_0}^{\text{EUR}}$ at time t_0. Thus, what we have here is a *spot purchase* of dollars at the rate e_{t_0}.

The synthetic is now fully described:

- Take an interbank loan in euros (Figure 3-8d).
- Using these euro funds, buy spot dollars (Figure 3-8f).
- Deposit these dollars in the interbank market (Figure 3-8e).

This portfolio would exactly replicate the currency forward, since by adding the cash flows in Figures 3-8d, 3-8e, and 3-8f, we recover exactly the cash flows generated by a currency forward shown in Figure 3-8a.

4.2.2. A Synthetic with T-Bills

We can also create a synthetic currency forward using Treasury-bill markets. In fact, let $B(t_0, t_1)^{\text{USD}}$ be the time-t_0 price of a default-free discount bond that pays USD100 at time t_1. Similarly, let $B(t_0, t_1)^{\text{EUR}}$ be the time-t_0 price of a default-free discount bond that pays EUR100 at time t_1. Then the cash flows in Figures 3-8d, 3-8e, and 3-8f can be reinterpreted so as to represent the following transactions:[4]

- Figure 3-8d is a *short* position in $B(t_0, t_1)^{\text{EUR}}$ where $1/F_{t_0}$ units of this security is borrowed and sold at the going market price to generate $B(t_0, t_1)^{\text{EUR}}/F_{t_0}$ euros.
- In Figure 3-8f, these euros are exchanged into dollars at the going exchange rate.
- In Figure 3-8e, the dollars are used to buy one dollar-denominated bond $B(t_0, t_1)^{\text{USD}}$.

At time t_1 these operations would amount to exchanging EUR $100/F_{t_0}$ against USD100, given that the corresponding bonds mature at par.

4.2.3. Which Synthetic Should One Use?

If synthetics for an instrument can be created in many ways, which one should a financial engineer use in hedging, risk management, and pricing? We briefly comment on this important question.

As a rule, a market practitioner would select the synthetic instrument that is most desirable according to the following attributes: (1) The one that *costs* the least. (2) The one that is most *liquid*, which, *ceteris paribus*, will, in general, be the one that costs the least. (3) The one that is most convenient for *regulatory* purposes. (4) The one that is most appropriate given *balance sheet* considerations. Of course, the final decision will have to be a compromise and will depend on the particular needs of the market practitioner.

4.2.4. Credit Risk

Section 4.2.1 displays a list of instruments that have similar cash flow patterns to loans and T-bills. The assumption of no-credit risk is a major reason why we could alternate between loans and T-bills in Sections 4.2.1 and 4.2.2. If credit risk were taken into account, the cash flows would be significantly different. In particular, for loans we would have to consider a diagram such as in Figure 3-13, whereas T-bills would have no default risks.

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[4] Disregard for the time being whether such liquid discount bonds exist in the desired maturities.

5.   Synthetics and Pricing

A major use of synthetic assets is in pricing. Everything else being the same, a replicating portfolio must have the same price as the original instrument. Thus, adding up the values of the constituent assets we can get the *cost* of forming a replicating portfolio. This will give the price of the original instrument once the market practitioner adds a proper margin.

In the present context, *pricing* means obtaining the unknowns in the currency forward, which is the forward exchange rate, F_{t_0} introduced earlier. We would like to determine a set of pricing equations which result in *closed-form* pricing formulas. Let us see how this can be done.

Begin with Figure 3-8f. This figure implies that the time-t_0 market values of $C_{t_0}^{\text{USD}}$ and $C_{t_0}^{\text{EUR}}$ should be the same. Otherwise, one party will not be willing to go through with the deal. This implies,

$$C_{t_0}^{\text{USD}} = C_{t_0}^{\text{EUR}} e_{t_0} \tag{3}$$

where e_{t_0} is the *spot* EUR/USD exchange rate. Replacing from equations (1) and (2):

$$F_{t_0}\left[\frac{100}{1 + L_{t_0}^{\text{USD}}(\frac{t_1-t_0}{360})}\right] = \left[\frac{100}{1 + L_{t_0}^{\text{EUR}}(\frac{t_1-t_0}{360})}\right] e_{t_0} \tag{4}$$

Solving for the forward exchange rate F_{t_0},

$$F_{t_0} = e_{t_0}\left[\frac{1 + L_{t_0}^{\text{USD}}(\frac{t_1-t_0}{360})}{1 + L_{t_0}^{\text{EUR}}(\frac{t_1-t_0}{360})}\right] \tag{5}$$

This is the well-known *covered interest parity* equation. Note that it expresses the "unknown" F_{t_0} as a function of variables that can be observed at time t_0. Hence, using the market quotes F_{t_0} can be numerically calculated at time t_0 and does not require *any* forecasting effort.[5]

The second synthetic using T-bills gives an alternative pricing equation. Since the values evaluated at the current exchange rate, e_t, of the two bond positions needs to be the same, we have

$$F_{t_0} B(t_0, t_1)^{\text{USD}} = e_{t_0} B(t_0, t_1)^{\text{EUR}} \tag{6}$$

Hence, the F_{t_0} priced *off the T-bill markets* will be given by

$$F_{t_0} = e_{t_0} \frac{B(t_0, t_1)^{\text{EUR}}}{B(t_0, t_1)^{\text{USD}}} \tag{7}$$

If the bond markets in the two currencies is as liquid as the corresponding deposits and loans, and if there is no credit risk, the F_{t_0} obtained from this synthetic will be very close to the F_{t_0} obtained from deposits.[6]

6.   A Contractual Equation

Once an instrument is replicated with a portfolio of other (liquid) assets, we can write a *contractual equation* and create new synthetics. In this section, we will obtain a contractual equation.

---

[5] In fact, bringing in a forecasting model to determine the F_{t_0} will lead to the wrong market price and may create arbitrage opportunities.

[6] Remember the important point that, in practice, both the liquidity and the credit risks associated with the synthetics could be significantly different. Then the calculated F_{t_0} would diverge.

In the next section, we will show several applications. This section provides a basic approach to constructing static replicating portfolios and hence is central to what will follow.

We have just created a synthetic for currency forwards. The basic idea was that a portfolio consisting of the following instruments:

{*Loan in EUR, Deposit of USD, spot purchase of USD against EUR*}

would generate the same cash flows, at the same *time* periods, with the same *credit risk* as the currency forward. This means that under the (unrealistic) assumptions of

1. No transaction costs
2. No bid-ask spreads
3. No credit risk
4. Liquid markets

we can write the equivalence between the related synthetic and the original instrument as a contractual equation that can conveniently be exploited in practice. In fact, the synthetic using the money market involved three contractual deals that can be summarized by the following "equation":

$$
\boxed{\begin{array}{l}\textbf{FX forward}\\ \text{Buy USD against}\\ \text{EUR}\end{array}} = \boxed{\begin{array}{l}\textbf{Loan}\\ \text{Borrow EUR at } t_0\\ \text{for maturity } t_1\end{array}} + \boxed{\begin{array}{l}\textbf{Spot Operation}\\ \text{Using the proceeds,}\\ \text{buy USD}\\ \text{against EUR}\end{array}} + \boxed{\begin{array}{l}\textbf{Deposit}\\ \text{Deposit USD at } t_0\\ \text{for maturity } t_1\end{array}} \quad (8)
$$

This operation can be applied to any two currencies to yield the corresponding FX forward.

The expression shown in Formula (8) is a *contractual equation*. The left-hand side contract leads to the same cash flows generated *jointly* by the contracts on the right-hand side. This does not necessarily mean that the monetary *value* of the two sides is always the same. In fact, one or more of the contracts shown on the right-hand side may not even *exist* in that particular economy and the markets may not even have the opportunity to put a price on them.

Essentially the equation says that the risk and cash flow attributes of the two sides are the same. If there is no credit risk, no transaction costs, and if the markets in *all* the involved instruments are liquid, we expect that *arbitrage* will make the values of the two sides of the contractual equation equal.

7.   Applications

The contractual equation derived earlier and the manipulation of cash flows that led to it may initially be thought of as theoretical constructs with limited practical application. This could not be further from the truth. We now discuss four examples that illustrate how the equation can be skillfully exploited to find solutions to practical, common problems faced by market participants.

7.1.   *Application 1: A Withholding Tax Problem*

We begin with a practical problem of withholding taxes on interest income. Our purpose is not to comment on the taxation aspects but to use this example to motivate uses of synthetic instruments.

The basic idea here is easy to state. If a government imposes withholding taxes on gains from a particular instrument, say a *bond*, and if it is possible to synthetically replicate this instrument, then the synthetic may not be subject to withholding taxes. If one learns how to do this, then the net returns offered to clients will be significantly higher—with, essentially, the same risk.

EXAMPLE:

Suppose an economy has imposed a withholding tax on interest income from government bonds. Let this withholding tax rate be 20%. The bonds under question have zero default probability and make no coupon payments. They mature at time-T and their time-t price is denoted by $B(t, T)$. This means that if

$$B(t, T) = 92 \tag{9}$$

one pays 92 dollars at time t to receive a bond with face value 100. The bond matures at time T, with the maturity value

$$B(T, T) = 100 \tag{10}$$

Clearly, the interest the bondholder has earned *will be given by*

$$100 - B(t, T) = 8 \tag{11}$$

But because of the withholding tax, the interest received *will only be* 6.4 :

$$Interest\ received = 8 - .2(8) = 6.4 \tag{12}$$

Thus the bondholder receives significantly less than what he or she earns.

We can immediately use the ideas put forward to form a synthetic for any discount bond using the contractual equation in formula (8). We discuss this case using two arbitrary currencies called Z and X. Suppose T-bills in both currencies trade actively in their respective markets. The contractual equation written in terms of T-bills gives

$$
\boxed{\begin{array}{l}\textbf{FX forward} \\ \text{Sell currency } Z \\ \text{against currency } X\end{array}} = \boxed{\begin{array}{l}\textbf{Short } Z\textbf{-} \\ \textbf{denominated} \\ \textbf{T-bill}\end{array}} + \boxed{\begin{array}{l}\textbf{Spot Operation} \\ \text{Buy currency } X \\ \text{with currency } Z\end{array}} + \boxed{\begin{array}{l}\textbf{Buy } X\textbf{-} \\ \textbf{denominated} \\ \textbf{T-bill}\end{array}} \tag{13}
$$

Manipulating this as an algebraic equation, we can transfer the Z-denominated T-bill to the left-hand side and group all other instruments on the right-hand side.

$$
-\boxed{\begin{array}{l}\textbf{Short } Z\textbf{-} \\ \textbf{denominated} \\ \textbf{T-bill}\end{array}} = -\boxed{\begin{array}{l}\textbf{FX forward} \\ \text{Sell } Z \text{ against } X\end{array}} + \boxed{\begin{array}{l}\textbf{Spot Operation} \\ \text{Buy currency } X \\ \text{with } Z\end{array}} + \boxed{\begin{array}{l}\textbf{Buy } X\textbf{-} \\ \textbf{denominated} \\ \textbf{T-bill}\end{array}}
$$

$$\tag{14}$$

Now, we change the negative signs to positive, which reverses the cash flows, and obtain a synthetic Z-denominated T-Bill:

$$
\begin{array}{c}
\boxed{\begin{array}{l}\textbf{Long } Z\text{-} \\ \textbf{denominated} \\ \textbf{T-bill}\end{array}}
=
\boxed{\begin{array}{l}\textbf{FX forward} \\ \text{Buy } Z \text{ against } X\end{array}}
+
\boxed{\begin{array}{l}\textbf{Spot Operation} \\ \text{Buy currency } X \\ \text{with } Z\end{array}}
+
\boxed{\begin{array}{l}\textbf{Buy } X\text{-} \\ \textbf{denominated} \\ \textbf{T-bill}\end{array}}
\end{array}
\qquad (15)
$$

Thus, in order to construct a synthetic for Z-denominated discount bonds, we first need to use money or T-bill markets of another economy where there is no withholding tax. Let the currency of this country be denoted by the symbol X. According to equation (15) we exchange Z's into currency X with a *spot* operation at an exchange rate e_{t_0}. Using the X's obtained this way we buy the relevant X-denominated T-bill. At the same time we *forward purchase Z's* for time t_1. The geometry of these operations is shown in Figure 3-9. We see that by adding the
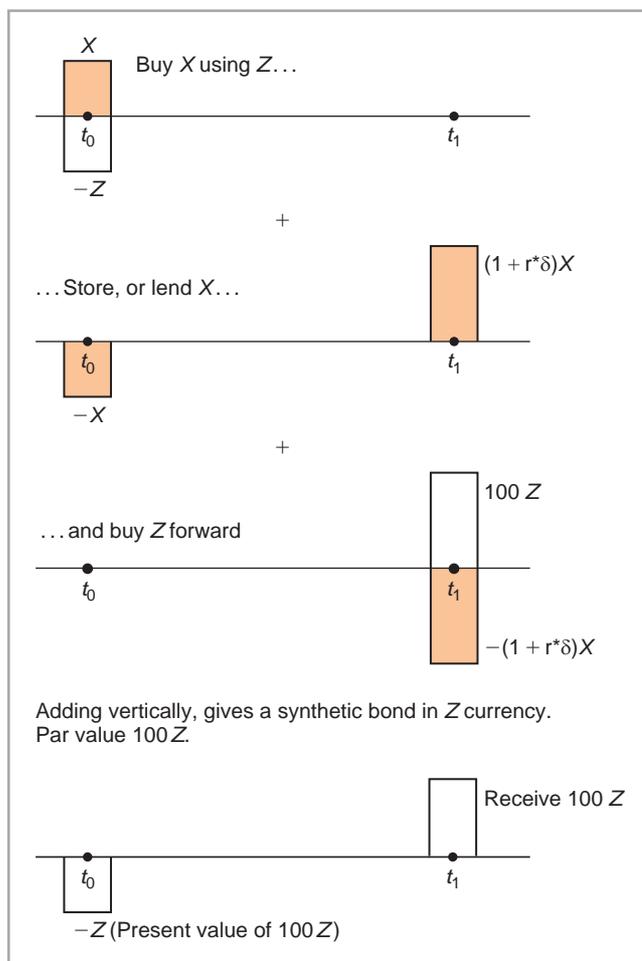

FIGURE 3-9

cash flows generated by the right-hand side operations, we can get exactly the cash flows of a T-bill in Z.

There is a simple logic behind these operations. Investors are taxed on Z-denominated bonds. So they use *another* country's markets where there is no withholding tax. They do this in a way that ensures the recovery of the needed Z's at time t_1 by buying Z forward. In a nutshell, this is a strategy of carrying funds over time using another currency as a *vehicle* while making sure that the *entry* and *exits* of the position are pinned down at time t_0.

7.2. Application 2: Creating Synthetic Loans

The second application of the contractual equation has already been briefly discussed in Chapter 1. Consider the following market event from the year 1997.

EXAMPLE:

Following the collapse of Hokkaido Takushoku Bank, the "Japanese premium," the extra cost to Japanese banks of raising money in the Eurodollar market increased last week in dramatic style. Japanese banks in the dollar deposit market were said to be paying around 40 bp over their comparable U.S. credits, against less than 30 bp only a week ago.

Faced with higher dollar funding costs, Japanese banks looked for an alternative source of dollar finance. Borrowing in yen and selling yen against the dollar in the spot market, they bought yen against dollars in the forward market, which in turn caused the U.S. dollar/yen forward rate to richen dramatically. (IFR, November 22, 1997)

Readers with no market experience may consider this episode difficult to understand.[7] Yet, the contractual equation in formula (8) can be used skillfully, to explain the strategy of Japanese banks mentioned in the example. In fact, what Japanese banks were trying to do was to create synthetic USD loans. The USD loans were either too expensive or altogether unavailable due to lack of *credit lines*. As such, the excerpt provides an excellent example of a use for synthetics.

We now consider this case in more detail. We begin with the contractual equation in formula (8) again, but this time write it for the USD/JPY exchange rate:

$$
\boxed{\begin{array}{l}\textbf{FX forward}\\ \text{Sell USD against}\\ \text{JPY for time } t_1\end{array}} = \boxed{\begin{array}{l}\textbf{Loan}\\ \text{Borrow USD with}\\ \text{maturity } t_1\end{array}} + \boxed{\begin{array}{l}\textbf{Spot Operation}\\ \text{Buy JPY pay USD}\\ \text{at } t_0\end{array}} + \boxed{\begin{array}{l}\textbf{Deposit}\\ \text{Deposit JPY for}\\ \text{maturity } t_1\end{array}}
$$

(16)

Again, we manipulate this like an algebraic equation. Note that on the right-hand side there is a loan contract. This is a genuine USD loan, and it can be isolated on the left-hand side by rearranging the right-hand side contracts. The loan would then be expressed in terms of a replicating portfolio.

---

[7] However, we clarify one point immediately: A basis point (bp) is one-hundredth of 1%. In other words, 1% equals 100bs.

$$
\boxed{\begin{array}{l}\textbf{Loan}\\ \text{Borrow USD with}\\ \text{maturity } t_1\end{array}} = \boxed{\begin{array}{l}\textbf{FX forward}\\ \text{Sell USD against}\\ \text{JPY for time } t_1\end{array}} - \boxed{\begin{array}{l}\textbf{Spot Operation}\\ \text{Buy JPY pay USD}\\ \text{at } t_0\end{array}} - \boxed{\begin{array}{l}\textbf{Deposit}\\ \text{Deposit JPY for}\\ \text{maturity } t_1\end{array}}
$$

(17)

Note that because we moved the deposit and the spot operation to the other side of the equality, signs changed. In this context, a deposit with a minus sign would mean reversing the cash flow diagrams and hence it becomes a loan. A spot operation with a minus sign would simply switch the currencies exchanged. Hence, the contractual equation can finally be written as

$$
\boxed{\textbf{USD loan}} = \boxed{\begin{array}{l}\textbf{FX Forward}\\ \text{Sell USD against}\\ \text{JPY for time } t_1\end{array}} + \boxed{\begin{array}{l}\textbf{Spot Operation}\\ \text{Buy USD against}\\ \text{JPY at } t_0\end{array}} + \boxed{\begin{array}{l}\textbf{A Loan}\\ \text{Borrow JPY for}\\ \text{maturity } t_1\end{array}}
$$

(18)

This contractual equation can be used to understand the previous excerpt. According to the quote, Japanese banks that were hindered in their effort to borrow Eurodollars in the interbank (Euro) market instead borrowed Japanese yen in the domestic market, which they used to buy (cash) dollars. But, at the same time, they sold dollars forward against yen in order to hedge their future currency exposure. Briefly, they created exactly the synthetic that the contractual equation implies on the right-hand side.

7.3. *Application 3: Capital Controls*

Several countries have, at different times, imposed restriction on capital movements. These are known as *capital controls*. Suppose we assume that a spot purchase of USD against the local currency X is *prohibited* in some country.

A financial engineer can construct a *synthetic spot operation* using the contractual relationship, since such spot operations were one of the constituents of the contractual equation shown in formula (8). Rearranging formula (8), we can write

$$
\boxed{\begin{array}{l}\textbf{Spot purchase} \text{ of}\\ \text{USD against } X\end{array}} = \boxed{\begin{array}{l}\textbf{FX-Forward}\\ \text{Sell } X \text{ against USD}\\ \text{for time } t_1\end{array}} + \boxed{\begin{array}{l}\textbf{Loan in USD}\\ \text{Borrow USD at } t_0.\end{array}} + \boxed{\begin{array}{l}\textbf{Deposit } X \text{ at } t_0\\ \text{for maturity } t_1\end{array}}
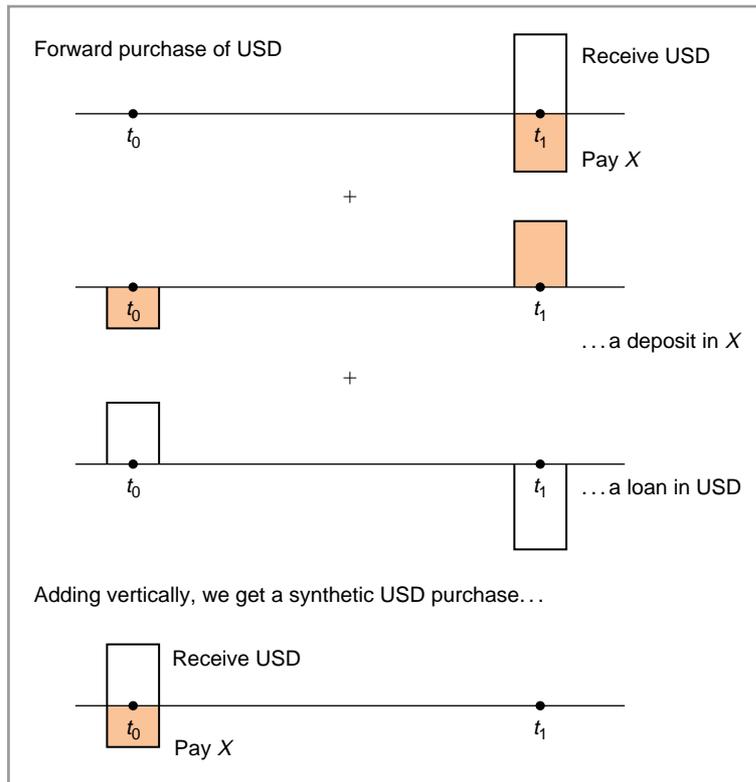$$

(19)

FIGURE 3-10

The right-hand side will be equivalent to a spot purchase of USD even when there are capital controls. Precursors of such operations were called *parallel loans* and were extensively used by businesses, especially in Brazil and some other emerging markets.[8] The geometry of this situation is shown in Figure 3-10.

7.4. Application 4: "Cross" Currencies

Our final example does not make use of the contractual equation in formula (8) directly. However, it is an interesting application of the notion of contractual equations, and it is appropriate to consider it at this point.

One of the simplest synthetics is the "cross rates" traded in FX markets. A cross currency exchange rate is a price that does not involve USD. The major "crosses" are EUR/JPY, EUR/CHF, GBP/EUR. Other "crosses" are relatively minor. In fact, if a trader wants to purchase Swiss francs in, for example, Taiwan, the trader would carry out two transactions instead of a single spot transaction. He or she would buy U.S. dollars with Taiwan dollars, and then sell the U.S. dollars against the Swiss franc. At the end, Swiss francs are paid by Taiwan dollars. Why would one go through two transactions instead of a direct purchase of Swiss francs in Taiwan? Because it is cheaper to do so, due to lower transaction costs and higher liquidity of the USD/CHF and USD/TWD exchange rates.

---

[8] One may ask the following question: If it is not possible to buy foreign currency in an economy, how can one borrow in it? The answer to this is simple. The USD borrowing is done with a foreign counterparty.
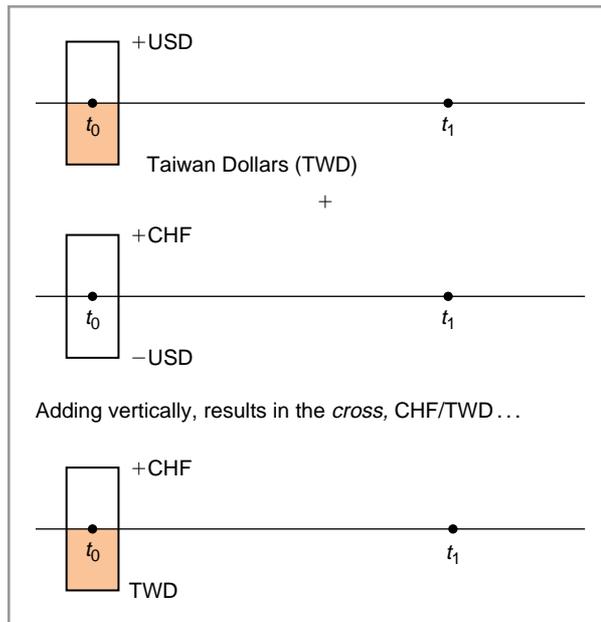
FIGURE 3-11

We can formulate this as a contractual equation:

Spot purchase of CHF using Taiwan dollars	=	Buy USD against Taiwan dollar	+	Sell USD against Swiss francs

(20)

It is easy to see why this contractual equation holds. Consider Figure 3-11. The addition of the cash flows in the top two graphs results in the elimination of the USD element, and one creates a synthetic "contract" of spot purchase of CHF against Taiwan dollars.

This is an interesting example because it shows that the price differences between the synthetic and the actual contract cannot always be exploited due to transaction costs, liquidity, and other rigidities such as the legal and organizational framework. It is also interesting in this particular case, that it is the synthetic instrument which turns out to be *cheaper*. Thus, before buying and selling an instrument, a trader should always try to see if there is a cheaper synthetic that can do the same job.

8. A "Better" Synthetic

In the previous sections we created two synthetics for forward FX-contracts. We can now ask the next question: Is there an optimal way of creating a synthetic? Or, more practically, can a trader buy a synthetic cheaply, and sell it to clients after adding a margin, and still post the smallest bid-ask spreads?
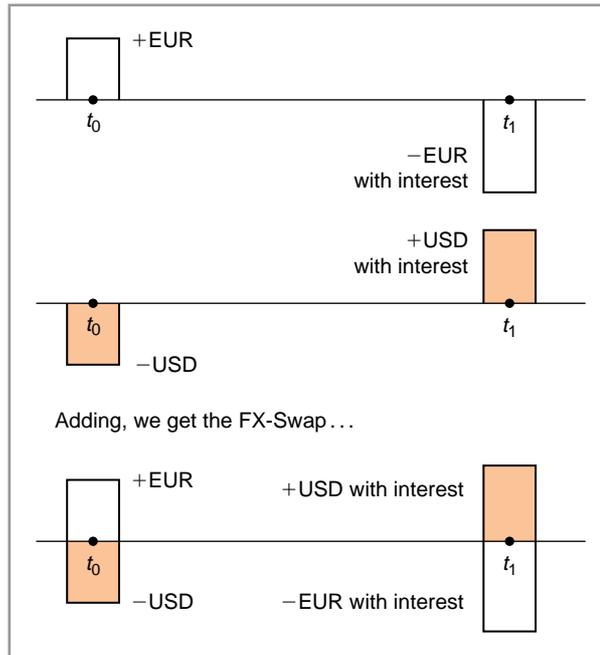
FIGURE 3-12

8.1. *FX-Swaps*

We can use the so-called *FX-swaps* and pay a single bid-ask spread instead of going through two separate bid-ask spreads, as is done in contractual equation (8). The construction of an FX-swap is shown in Figure 3-12.

According to this figure there are at least two ways of looking at a FX-swap. The FX-swap is made of a money market deposit and a money market loan in different currencies written on the same "ticket." The second interpretation is that we can look at a FX-swap as if the two counterparties *spot* purchase and *forward* sell two currencies against each other, again on the same deal ticket.

When combined with a spot operation, FX-swaps duplicate forward currency contracts easily, as seen in Figure 3-13. Because they are swaps of a deposit against a loan, interest rate differentials will play an important role in FX-swaps. After all, one of the parties will be giving away a currency that can earn a higher rate of interest and, as a result, will demand compensation for this "loss." This compensation will be returned to him or her as a proportionately higher payment at time t_1. The parties must exchange *different* amounts at time t_1, as compared to the original exchange at t_0.

8.1.1. Advantages

Why would a bank prefer to deal in FX swaps instead of outright forwards? This is an important question from the point of view of financial engineering. It illustrates the advantages of spread products.

FX-swaps have several advantages over the synthetic seen earlier. First of all, FX-swaps are interbank instruments and, normally, are not available to clients. Banks deal with each other every day, and thus relatively little counterparty risk exists in writing such contracts. In liquid
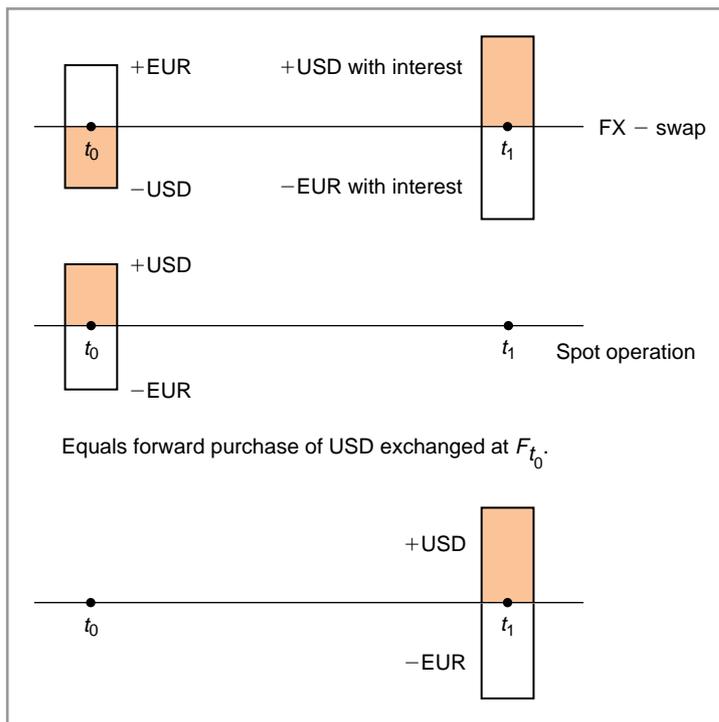
FIGURE 3-13

markets, the implied bid-ask spread for synthetics constructed using FX-swaps will be smaller than the synthetic constructed from deposits and loans, or T-bills for that matter.

The second issue is liquidity. How can a market participant borrow and lend in both currencies without moving prices? A FX-swap is again a preferable way of doing this. With a FX-swap, traders are not *buying* or *selling* deposits, rather they are exchanging them.

The final advantage of FX-swaps reside in their balance sheet effects, or the lack thereof. The synthetic developed in Figure 3-8 leads to increased assets and liabilities. One borrows new funds and lends them. Such transactions may lead to new credit risks and new capital requirements. FX-swaps are off-balance sheet items, and the synthetic in Figure 3-13 will have minor balance sheet effects.

8.1.2. Quotation Conventions

Banks prefer to quote *swap* or *forward points* instead of quoting the *outright* forward exchange rate. The related terminology and conventions are illustrated in the following example:

EXAMPLE:

Suppose outright forward EUR/USD quotes are given by

Bid	Ask
1.0210	1.0220

and that the spot exchange rate quotes are as

Bid	Ask
1.0202	1.0205

Then, instead of the outright forward quotes, traders prefer to quote the forward points *obtained by subtracting the corresponding spot rate from the outright forward*

Bid	Ask
8	15

In reality forward points are determined directly from equation (5) or (7).

Market conventions sometimes yield interesting information concerning trading activity and the forward FX quotes is a case in point. In fact, there is an important advantage to quoting swap points over the outright forward quotes. This indicates a subtle aspect of market activity. A quote in terms of forward points will essentially be independent of spot exchange rate movements and will depend only on interest rate differentials. An outright forward quote, on the other hand, will depend on the spot exchange rate movements as well. Thus, by quoting forward points, market professionals are essentially *separating* the risks associated with interest rate differentials and spot exchange rate movements respectively. The exchange rate risk will be left to the spot trader. The forward-FX trader will be trading the risk associated with interest rate differentials only.

To see this better, we now look at the details of the argument. Let F_{t_0} and e_{t_0} be time-t_1 forward and time-t_0 spot exchange rates respectively as given by equation (5). Using the expression in equation (5) and ignoring the bid-ask spreads, we can write approximately,

$$F_{t_0} - e_{t_0} \cong (r_{t_0}^d - r_{t_0}^f) \left(\frac{t_1 - t_0}{360} \right) e_{t_0} \qquad (21)$$

where the $r_{t_0}^d, r_{t_0}^f$ are the relevant interest rates in domestic and foreign currencies, respectively.[9] Taking partial derivatives this equation gives

$$\partial(F_{t_0} - e_{t_0}) \cong (r_{t_0}^d - r_{t_0}^f) \left(\frac{t_1 - t_0}{360} \right) \partial e_{t_0}$$

$$\cong 0 \qquad (22)$$

If the *daily* movement of the spot rate e_{t_0} is small, the right hand side will be negligible. In other words, the forward swap quotes would not change for normal daily exchange rate movements, if interest rates remain the same and *as long as* exchange rates are quoted to four decimal places. The following example illustrates what this means.

EXAMPLE:

Suppose the relevant interest rates are given by

$$r_{t_0}^d = .03440 \qquad (23)$$
$$r_{t_0}^f = .02110 \qquad (24)$$

---

[9] This assumes a day count basis of 360 days. If one or both of the interest rates have a 365-day convention, this expression needs to be adjusted accordingly.

where the domestic currency is euro and the foreign currency is USD. If the EUR/USD exchange rate has a daily volatility of, say, .01% a day, which is a rather significant move, then, for FX-swaps with 3-month maturity we have the following change in forward points:

$$\partial(F_{t_0} - e_{t_0}) = .01330 \left(\frac{90}{360} \right) 0.0100$$

$$= .00003325 \qquad (25)$$

which, in a market that quotes only four decimal points, will be negligible.

Hence, forward points depend essentially on the interest rate differentials. This "separates" exchange rate and interest rate risk and simplifies the work of the trader. It also shows that forward FX contracts can be interpreted as if they are "hidden" interest rate contracts.

9. Futures

Up to this point we considered forward contracts written on currencies only. These are OTC contracts, designed according to the needs of the clients and negotiated between two counterparties. They are easy to price and almost costless to purchase.

Futures are different from forward contracts in this respect. Some of the differences are minor; others are more important, leading potentially to significantly different forward and futures prices on the same underlying asset with identical characteristics. Most of these differences come from the design of futures contracts. Futures contracts need to be homogeneous to increase liquidity. The way they *expire* and the way *deliveries* are made will be clearly specified, but will still leave some options to the players. Forward contracts are initiated between two specific parties. They can state exactly the delivery and expiration conditions. Futures, on the other hand, will leave some room for last-minute adjustments and these "options" may have market value.

In addition, futures contracts are always *marked to market*, whereas this is a matter of choice for forwards. Marking to market may significantly alter the implied cash flows and result in some moderate convexities.

To broaden the examination of futures and forwards in this section, we concentrate on *commodities* that are generally traded via futures contracts in organized exchanges. Let S_t denote the spot price on an underlying commodity and f_t be the *futures price* quoted in the exchange.

9.1. *Parameters of a Futures Contract*

We consider two contracts in order to review the main parameters involved in the design of a futures. The key point is that most aspects of the transaction need to be pinned down to make a homogeneous and liquid contract. This is relatively easy and straightforward to accomplish in the case of a relatively standard commodity such as soybeans.

EXAMPLE: *CBOT Soybeans Futures*

1. **Contract size.** 5000 bushels.
2. **Deliverable grades.** No. 2 yellow at par, No. 1 yellow at 6 cents per bushel over contract price, and No. 3 yellow at 6 cents per bushel under contract price.

(Note that in case a trader accepts the delivery, a special type of soybeans will be delivered to him or her. The trader may, in fact, procure the same quantity under better conditions from someone else. Hence, with a large majority of cases, futures contracts do not end with delivery. Instead, the position is unwound with an opposite transaction sometime before expiration.)

3. **Tick size.** Quarter-cent/bushel ($12.50/contract).
4. **Price quote.** Cents and quarter-cent/bushel.
5. **Contract months.** September, November, January, March, May, July, and August. (Clearly, if the purpose behind a futures transaction is delivery, then forward contracts with more flexible delivery dates will be more convenient.)
6. **Last trading day.** The business day prior to the 15th calendar day of the contract month.
7. **Last delivery day.** Second business day following the last trading day of the delivery month.
8. **Trading hours.** Open outcry: 9:30 a.m. to 1:15 p.m. Chicago time, Monday through Friday. Electronic, 8:30 p.m. to 6:00 a.m. Chicago time, Sunday through Friday. Trading in expiring contracts closes at noon on the last trading day.
9. **Daily price limit.** 50 cents/bushel ($2500/contract) above or below the previous day's settlement price. No limit in the spot month. (Limits are lifted two business days before the spot month begins.)

A second example is from financial futures. Interest rate futures constitute some of the most liquid instruments in all markets. They are, again, homogenized contracts and will be discussed in the next chapter.

EXAMPLE: *LIFFE 3-Month Euro Libor Interest-Rate Futures*

1. **Unit of trading.** Euro 1,000,000.
2. **Delivery months.** March, June, September, and December. June 2003 is the last contract month available for trading.
3. **Price quotes.** 100 minus rate of interest. (Note that prices are quoted to three decimal places. This means that the British Bankers Association (BBA) Libor will be rounded to three decimal places and will be used in settling the final value of the contract.)
4. **Minimum price movement.** (Tick size and value) 0.005 (12.50).
5. **Last trading day.** Two business days prior to the third Wednesday of the delivery month.
6. **Delivery day.** First business day after the last trading day.
7. **Trading hours.** 07:00 to 18:00.

Such Eurocurrency futures contracts will be discussed in the next chapter and will be revisited several times later. In particular, one aspect of the contract that has not been listed among the parameters noted here has interesting financial engineering implications. Eurocurrency futures have a *quotation convention* that implies a *linear* relationship between the forward interest rate and the price of the futures contract. This is another example of the fact that conventions are indeed important in finding the right solution to a financial engineering problem.

One final, but important point. The parameters of futures contracts are sometimes revised by Exchanges; hence the reader should consider the information provided here simply as examples and check the actual contract for specifications.

9.2. Marking to Market

We consider the cash flows generated by a futures contract and compare them with the cash flows on a forward contract on the same underlying. It turns out that, unlike forwards, the effective maturity of a futures position is, in fact, *1 day*. This is due to the existence of *marking to market* in futures trading. The position will be marked to market in the sense that every night the exchange will, in effect, close the position and then reopen it at the new settlement price. It is best to look at this with a precise example. Suppose a futures contract is written on one unit of a commodity with spot price S_t. Suppose t is a Monday and that the expiration of the contract is within *3* trading days:

$$T = t + 3 \tag{26}$$

Suppose further that during these days, the *settlement prices* follow the trajectory

$$f_t > f_{t+1} > f_{t+2} = f_{t+3} \tag{27}$$

What cash flows will be generated by a *long* position in one futures contract if at expiration date T the position is closed by taking the offsetting position?[10]

The answer is shown in Figure 3-14. Marking to market is equivalent to forcing the long (short) position holder to close his position at that day's settlement price and reopen it again at the same price. Thus, at the end of the first trading day after the trade, the futures contract that was "purchased" at f_t will be "sold" at the f_{t+1} shown in equation (27) for a loss:

$$f_{t+1} - f_t < 0 \tag{28}$$

Similarly, at the end of the second trading day, marking to market will lead to another loss:

$$f_{t+2} - f_{t+1} < 0 \tag{29}$$

This is the case since, according to trajectory in equation (27), prices have declined again. The expiration date will see no further losses, since, by chance, the final settlement price is the same as the previous day's settlement.

In contrast, the last portion of Figure 3-14 shows the cash flows generated by the forward prices F_t. Since there is no marking to market (in this case), the only capital loss occurs at the expiration of the contract. Clearly, this is a very different cash flow pattern.

9.3. Cost of Carry and Synthetic Commodities

What is the *carry cost* of a position? We will answer this question indirectly. In fact, ignoring the mark to market and other minor complications, we first apply the contractual equation developed earlier to create *synthetic commodities*.

For example, suppose S_t represents spot coffee, which is the underlying asset for a futures contract with price f_t and expiration date T, $t_0 < T$. How can we create a synthetic for this contract? The answer is quite similar to the case of currencies. Using the same logic, we can

---

[10] Instead of taking the offsetting position and cancelling out any obligations with respect to the clearinghouse, the trader could choose to accept delivery.

write a contractual equation:

$$
\boxed{\begin{array}{l}\text{Long coffee futures}\\ \text{expiration } T\end{array}} = \boxed{\begin{array}{l}\text{A Loan}\\ \text{Borrow}\\ \text{USD at } t_0 \text{ for}\\ \text{maturity } T\end{array}} + \boxed{\begin{array}{l}\text{Spot operation}\\ \text{Buy 1 unit of spot}\\ \text{coffee for } S_{t_0}\end{array}} + \boxed{\begin{array}{l}\text{Store the coffee at}\\ \text{a cost } q_{t_0} \text{ a day}\\ \text{until } T\end{array}} \quad (30)
$$

We can use this equation to obtain two results. First, by rearranging the contracts, we create a synthetic spot:

$$
\boxed{\begin{array}{l}\text{Spot operation}\\ \text{Buy one unit of spot}\\ \text{coffee for } S_{t_0}\end{array}} = - \boxed{\begin{array}{l}\text{A Loan}\\ \text{Borrow}\\ \text{USD at } t_0 \text{ for}\\ \text{maturity } T\end{array}} + \boxed{\begin{array}{l}\text{Long coffee futures}\\ \text{Expiration } T\end{array}} - \boxed{\begin{array}{l}\text{Store the coffee at}\\ \text{a cost } q_{t_0} \text{ a day}\\ \text{until } T\end{array}}
$$

$$(31)$$

In other words after changing signs, we need to *borrow* one unit of coffee, make a deposit of S_{t_0} dollars, and go long a coffee futures contract. This will yield a synthetic spot.

Second, the contractual equation can be used in pricing. In fact, the contractual equation gives the *carry cost* of a position. To see this first note that according to equation (30)
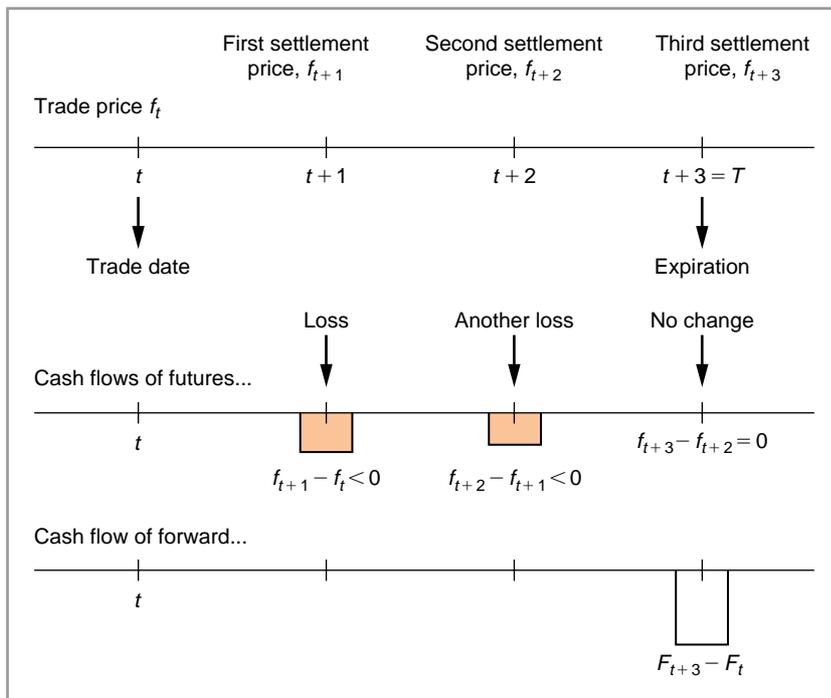

FIGURE 3-14

the value of the synthetic is the same as the value of the original contract. Then, we must have

$$f_{t_0} = (1 + r_{t_0}\delta)S_{t_0} + q_{t_0}(T - t_0) \tag{32}$$

where δ is the factor of days' adjustment to the interest rate denoted by the symbol r_{t_0}.

If storage costs are expressed as a percentage of the price, at an annual rate, just like the interest rates, this formula becomes

$$f_{t_0} = (1 + r_{t_0}\delta + q_{t_0}\delta)S_{t_0} \tag{33}$$

According to this, the more distant the expiration of the contracts is, the higher its price. This means that futures term structures would normally be upward sloping as shown in Figure 3-15. Such curves are said to be in *contango.* For some commodities, storage is either not possible (e.g., due to seasons) or prohibitive (e.g., crude oil). The curve may then have a negative slope and is said to be in *backwardation.* Carry cost is the interest plus storage costs here.

9.3.1.   A Final Remark

There are no upfront payments but buying futures or forward contracts is not costless. Ignoring any guarantees or *margins* that may be required for taking futures positions, taking forward or futures positions *does* involve a cost. Suppose we consider a storable commodity with spot price
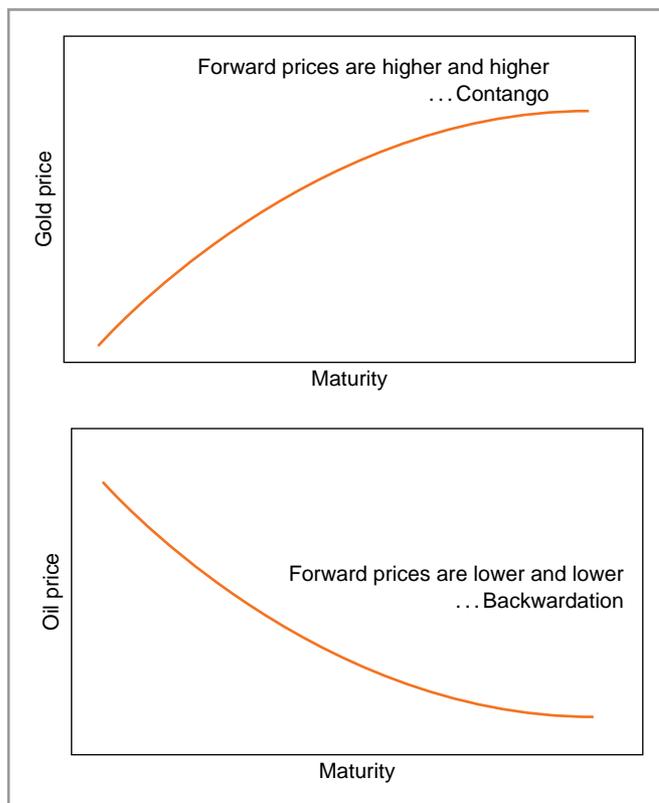

FIGURE 3-15

P_{t_0}. Let the forward price be denoted by $P_{t_0}^f$. Finally, suppose storage costs and all such effects are zero. Then the futures price is given by

$$P_{t_0}^f = (1 + r_{t_0}\delta)P_{t_0} \tag{34}$$

where the r_{t_0} is the appropriate interest rate that applies for the trader, and where δ is the time to expiration as a proportion of a year.

Now, suppose the spot price remains the *same* during the life of the contract. This means that the difference

$$P_{t_0}^f - P_{t_0} = r_{t_0}\delta P_{t_0} \tag{35}$$

is the cost of taking this position. Note that *this is as if we had borrowed P_{t_0} dollars for a "period" δ in order to carry a long position*. Yet there has been no exchange of principals. In the case of a default, no principal will be lost.

10. Conventions for Forwards

Forwards in foreign currencies have special quotation conventions. As mentioned earlier, in discussing FX-swaps, markets do not quote outright forward rates, but the so-called *forward points*. This is the difference between the forward rate found using the pricing equation in formula (21) and the spot exchange rate:

$$F_{t_0} - e_{t_0} \tag{36}$$

They are also called "pips" and written as bid/ask. We give an example for the way forward points are quoted and used.

EXAMPLE:

Suppose the spot and forward rate quotes are as follows:

EUR/USD	Bid	Ask
Spot	0.8567	0.8572
1yr	−28.3	−27.3
2yr	44.00	54.00

From this table we can calculate the outright forward exchange rate F_{t_0}.

For year 1, subtract the negative *pips in order to get the outright forward rates:*

$$\left(0.8567 - \frac{28.3}{10000}\right) \Big/ \left(0.8572 - \frac{27.3}{10000}\right) \tag{37}$$

For year 2, the quoted pips are positive. *Thus, we add the positive points to get the outright forward rates:*

$$\left(0.8567 + \frac{44}{10000}\right) \Big/ \left(0.8572 + \frac{54}{10000}\right) \tag{38}$$

Forward points give the amount needed to adjust the spot rate in order to obtain the outright forward exchange rate. Depending on the market, they are either added to or subtracted from the spot exchange rate. We should discuss briefly some related conventions.

There are two cases of interest. First, suppose we are given the following forward point quotes (second row) and spot rate quotes (first row) for EUR/USD:

Bid	*Ask*
1.0110	1.0120
12	16

Next note that the forward point listed in the "bid" column is lower than the forward point listed in the "ask" column. If forward point quotes are presented this way, then the points will be *added* to the last two digits of the corresponding spot rate.

Thus, we will obtain

$$\text{Bid forward outright} = 1.0110 + .0012 = 1.0122 \tag{39}$$
$$\text{Ask forward outright} = 1.0120 + .0016 = 1.0136 \tag{40}$$

Note that the bid-ask spread on the forward outright will be greater than the bid-ask spread on the spot.

Second, suppose, we have the following quotes:

Bid	*Ask*
1.0110	1.0120
23	18

Here the situation is reversed. The forward point listed in the "bid" column is greater than the forward point listed in the "ask" column. Under these conditions, the forward points will be *subtracted* from the last two digits of the corresponding spot rate. Thus, we will obtain

$$\text{Bid forward outright} = 1.0110 - .0023 = 1.0087 \tag{41}$$
$$\text{Ask forward outright} = 1.0120 - .0018 = 1.0102 \tag{42}$$

Note that the bid-ask spread on the forward outright will again be greater than the bid-ask spread on the spot. This second case is due to the fact that sometimes the minus sign is ignored in quotations of forward points.

11. Conclusions

This chapter has developed two main ideas. First, we considered the engineering aspects of future and forward contracts. Second, we developed our first contractual equation. This equation was manipulated to obtain synthetic loans, synthetic deposits, and synthetic spot transactions. A careful use of such contractual equations may provide useful techniques that are normally learned only after working in financial markets.

Before concluding, we would like to emphasize some characteristics of forward contracts that can be found in other swap-type derivatives as well. It is these characteristics that make these contracts very useful instruments for market practitioners.

First, at the time of initiation, the forward (future) contract did not require any initial *cash payments*. This is a convenient property if our business is trading contracts continuously during the day. We basically don't have to worry about "funding" issues.

Second, because forward (future) contracts have zero initial value, the position taker does not have anything to put on the balance sheet. The trader did not "*buy*" or "*sell*" something tangible. With a forward (futures) contract, the trader has simply taken a position. So these instruments are *off-balance sheet* items.

Third, forward contracts involve an exchange at a *future date*. This means that if one of the counterparties "defaults" before that date, the damage will be limited, since no principal amount was extended. What is at risk is simply any capital gains that may have been earned.

Suggested Reading

Futures and forward markets have now been established for a wide range of financial contracts, commodities, and services. This chapter dealt only with basic engineering aspects of such contracts, and a comprehensive discussion of futures was avoided. In the next chapter, we will discuss interest rate forwards and futures, but still many instruments will not be touched upon. It may be best to go over a survey of existing futures and forward contracts. We recommend two good introductory sources. The first is the Foreign Exchange and Money Markets, an introductory survey prepared by Reuters and published by Wiley. The second is the Commodities Trading Manual published by CBOT. **Hull** *(2002),* **Das** *(1994), and* **Wilmott** *(2000) are among the best sources for a detailed analysis of forward and futures contracts.*

Exercises

1. On March 3, 2000, the Financial Accounting Standards Board, a crucial player in financial engineering problems, published a series of important new proposals concerning the accounting of certain derivatives. It is known as Statement 133 and affects the daily lives of risk managers and financial engineers significantly. One of the treasurers who is affected by the new rules had the following comment on these new rules:

   Statement 133 in and of itself will make it a problem from an accounting point of view to do swaps. The amendment does not allow for a distinction to be made between users of aggressive swap hedges and those involved in more typical swaps. According to IFR this treasurer has used synthetic swaps *to get around [the FAS 133].*[11]

   (a) Ignoring the details of swaps as an instrument, what is the main point in FAS 133 that disturbs this market participant?
   (b) How does the treasurer expect to get around this problem by constructing synthetics?

2. In this question we consider a gold miner's hedging activities.

   (a) What is the natural position of a gold miner? Describe using payoff diagrams.
   (b) How would a gold miner hedge her position if gold prices are expected to drop steadily over the years? Show using payoff diagrams.
   (c) Would this hedge ever lead to losses?

3. Today is March 1, 2004. The day-count basis is actual/365. You have the following contracts on your FX-book.
   CONTRACT A: On March 15, 2004, you will sell 1,000,000 EUR at a price F_t^1 dollars per EUR.
   CONTRACT B: On April 30, 2004, you will buy 1,000,000 EUR at a price F_t^2 dollars per EUR.

   (a) Construct *one* synthetic equivalent of each contract.
   (b) Suppose the spot EUR/USD is 1.1500/1.1505. The USD interest rates for loans under 1 year equal 2.25/2.27, and the German equivalents equal 2.35/2.36. Calculate the F_t^i numerically.
   (c) Suppose the forward points for F_t^1 that we observe in the markets is equal to 10/20. How can an arbitrage portfolio be formed?

4. Consider the following instruments and the corresponding quotes. Rank these instruments in increasing order of their yields.

Instrument	Quote
30-day U.S. T-bill	5.5
30-day UK T-bill	5.4
30-day ECP	5.2
30-day interbank deposit USD	5.5
30-day U.S. CP	5.6

[11] IFR, Issue 1325.

(a) You purchase a ECP (Euro) with the following characteristics

Value date	July 29, 2002
Maturity	September 29, 2002
Yield	3.2%
Amount	10,000,000 USD

What payment do you have to make?

CASE STUDY: HKMA and the Hedge Funds, 1998

The Hong Kong Monetary Authority (HKMA) has been in the news because of you and your friends, hedge funds managers. In 1998, you are convinced of the following:

1. The HK\$ is overvalued by about 20% against the USD.
2. Hong Kong's economy is based on the real estate industry.
3. High interest rates cannot be tolerated by property developers (who incidentally are among Hong Kong's biggest businesses) and by the financial institutions.
4. Hong Kong's economy has entered a recession.

You decide to speculate on Hong Kong's economy with a "double play" that is made possible by the mechanics of the currency board system. You will face the HKMA as an adversary during this "play."

You are provided some background readings. You can also have the descriptions of various futures contracts that you may need for your activities as a hedge fund manager. Any additional data that you need should be searched for in the Internet. Answer the following questions:

1. What is the rationale of your double-play strategy?
2. In particular, how are HIBOR, HSI, and HSI futures related to each other?
3. Display your position explicitly using precise futures contract data.
4. How much will your position cost during 1 year?
5. How do you plan to roll your position over?
6. Looking back, did Hong Kong drop the peg?

Hedge Funds Still Bet the Currency's Peg Goes

HONG KONG–The stock market continued to rally last week in the belief the government is buying stocks to drive currency speculators out of the financial markets, though shares ended lower on Friday on profit-taking.

Despite the earlier rally, Hong Kong's economy still is worsening; the stock market hit a five-year low two weeks ago, and betting against the Hong Kong dollar is a cheap and easy wager for speculators.

The government maintains that big hedge funds that wager huge sums in global markets had been scooping up big profits by attacking both the Hong Kong dollar and the stock market.

Under this city's pegged-currency system, when speculators attack the Hong Kong dollar by selling it, that automatically boosts interest rates. Higher rates lure more investors to park their money in Hong Kong, boosting the currency. But they also slam the stock market because rising rates hurt companies' abilities to borrow and expand.

Speculators make money in a falling stock market by short-selling shares—selling borrowed shares in expectation that their price will fall and that the shares can be replaced more cheaply. The difference is the short-seller's profit.

"A lot of hedge funds which operate independently happen to believe that the Hong Kong dollar is overvalued" relative to the weak economy and to other Asian currencies, said Bill Kaye, managing director of hedge-fund outfit Pacific Group Ltd. Mr. Kaye points to Singapore where, because of the Singapore dollar's depreciation in the past year, office rents are now 30% cheaper than they are in Hong Kong, increasing the pressure on Hong Kong to let its currency fall so it can remain competitive.

Hedge funds, meanwhile, "are willing to take the risk they could lose money for some period," he said, while they bet Hong Kong will drop its 15-year-old policy of pegging the local currency at 7.80 Hong Kong dollars to the U.S. dollar.

These funds believe they can wager hundreds of millions of U.S. dollars with relatively little risk. Here's why: If a hedge fund bets the Hong Kong dollar will be toppled from its peg, it's a one-way bet, according to managers of such funds. That's because if the local dollar is dislodged from its peg, it is likely only to fall. And the only risk to hedge funds is that the peg remains, in which case they would lose only their initial cost of entering the trade to sell Hong Kong dollars in the future through forward contracts.

That cost can be low, permitting a hedge fund to eat a loss and make the same bet all over again. When a hedge fund enters a contract to sell Hong Kong dollars in, say, a year's time, it is committed to buying Hong Kong dollars to exchange for U.S. dollars in 12 months. If the currency peg holds, the cost of replacing the Hong Kong dollars it has sold is essentially the difference in 12-month interest rates between the U.S. and Hong Kong.

On Thursday, that difference in interbank interest rates was about 6.3 percentage points. So a fund manager making a US$1 million bet Thursday against the Hong Kong dollar would have paid 6.3%, or US$63,000.

Whether a fund manager wanted to make that trade depends on the odds he assigned to the likelihood of the Hong Kong dollar being knocked off its peg and how much he expected it then to depreciate.

If he believed the peg would depreciate about 30%, as a number of hedge-fund managers do, then it would have made sense to enter the trade if he thought there was a one-in-four chance of the peg going in a year. That's because the cost of making the trade—US$63,000—is less than one-fourth of the potential profit of a 30% depreciation, or US$300,000. For those who believe the peg might go, "it's a pretty good trade," said Mr. Kaye, the hedge-fund manager. He said that in recent months he hasn't shorted Hong Kong stocks or the currency.
Wall Street Journal, August 24, 1998.

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